# A no-go result for QBism

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**Abstract**

In QBism the wave function does not represent an element of physical reality external to the agent, but represent an agent's personal probability assignments, reflecting his subjective degrees of belief about the future content of his experience. In this paper, I argue that this view of the wave function is not consistent with protective measurements. The argument does not rely on the realist assumption of the $\psi$-ontology theorems, namely the existence of the underlying ontic state of a quantum system.

QBism is a new interesting approach to understanding quantum mechanics (Caves, Fuchs and Schack, 2002, 2007; Fuchs, Mermin and Schack, 2013; Fuchs and Stacey, 2016). It has received more and more attention in recent years (see, e.g. Mermin, 2014). In QBism the wave function represents an agent's personal probability assignments, reflecting his subjective degrees of belief about the future content of his experience such as the result of a measurement. Since QBism admits no agent-independent elements of physical reality that determine either measurement outcomes or probabilities of measurement outcomes, it is still consistent with the $\psi$-ontology theorems, which can be proved only when assuming the existence of the underlying ontic state of a quantum system (Pusey, Barrett and Rudolph, 2012; Colbeck and Renner, 2012; Hardy, 2013). However, it has been debated whether QBism provides a fully successful new framework for understanding quantum mechanics (Timpson, 2008; Bacciagaluppi 2014; Norsen, 2016; McQueen, 2017; Earman, 2019). In this paper, I will argue that the interpretation of the wave function according to QBism is not consistent with protective measurements. The argument does not rely on the realist assumption of the existence of the underlying ontic state of a quantum system.

Protective measurement (PM) is a method to measure the expectation value of an observable on a single quantum system (Aharonov and Vaidman, 1993; Aharonov, Anandan and Vaidman, 1993; Vaidman, 2009; Gao, 2015; Piacentini et al, 2017). For a conventional projective measurement, the measurement result will be in general random, being an eigenvalue of the measured observable with a probability in accordance with the Born rule, and the expectation value of the observable can be obtained only as the statistical average of eigenvalues for an ensemble of identically prepared systems. By contrast, during a PM the wave function of the measured system is protected by an appropriate procedure so that it keeps unchanged during the measurement.[1] Then, by the deterministic Schrödinger evolution that is independent of the Born rule, the measurement result will be definite, being the expectation value of the measured observable, even if the system is initially not in an eigenstate of the observable.

This result can be seen clearly from the following simple derivation. As for a projective measurement, the interaction Hamiltonian for measuring an observable $A$ is given by the usual form $H_I = g(t)PA$, where $g(t)$ is the time-dependent coupling strength of the interaction, which is a smooth function normalized to $\int_0^T g(t)dt = 1$ during the measurement interval $T$, and $g(0) = g(T) = 0$, and $P$ is the conjugate momentum of the pointer variable $X$. When the wave function of the measured system is protected to keep unchanged during the measurement, the evolution of the wave function of the combined system is

$$|\psi(0)\rangle\,|\phi(0)\rangle \rightarrow |\psi(t)\rangle\,|\phi(t)\rangle\,, t > 0, \tag{1}$$

where $|\phi(0)\rangle$ and $|\phi(t)\rangle$ are the wave functions of the measuring device at instants 0 and $t$, respectively, $|\psi(0)\rangle$ and $|\psi(t)\rangle$ are the wave functions of the measured system at instants 0 and $t$, respectively, and $|\psi(t)\rangle$ is the same as $|\psi(0)\rangle$ up to an overall phase during the measurement interval $[0, T]$. Then we have

$$\begin{aligned} \frac{d}{dt}\langle\psi(t)\phi(t)|X\,|\psi(t)\phi(t)\rangle &= \frac{1}{i\hbar}\langle\psi(t)\phi(t)|[X, H_I]\,|\psi(t)\phi(t)\rangle \\ &= g(t)\langle\psi(0)|A\,|\psi(0)\rangle\,, \end{aligned} \tag{2}$$

Note that the momentum expectation value of the pointer is zero at the initial instant and the free evolution of the pointer conserves it. This further

[1] Note that the protection requires that some information about the measured system should be known before a PM, and PMs cannot measure an arbitrary unknown wave function. In some cases, the information may be very little. For example, we only need to know that a quantum system such as an electron is in the ground state of an external potential before we make PMs on the system to find its wave function, no matter what form the external potential has.

leads to

$$\langle \phi(T)|X\,|\phi(T)\rangle - \langle \phi(0)|X\,|\phi(0)\rangle = \langle \psi(0)|A\,|\psi(0)\rangle\,, \tag{3}$$

which means that the shift of the center of the pointer wave packet is the expectation value of $A$ in the initial wave function of the measured system. This clearly demonstrates that the result of a measurement of an observable on a system, which does not change the wave function of the system, is the expectation value of the measured observable in the wave function of the measured system.

Since the wave function can be reconstructed from the expectation values of a sufficient number of observables, the wave function of a single quantum system can be measured by a series of PMs. Let the explicit form of the measured wave function at a given instant $t$ be $\psi(x)$, and the measured observable $A$ be (normalized) projection operators on small spatial regions $V_n$ having volume $v_n$:

$$A = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{4}$$

A PM of $A$ then yields

$$\langle A\rangle = \frac{1}{v_n}\int_{V_n} |\psi(x)|^2 dv, \tag{5}$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region $V_n$. Similarly, we can measure another observable $B = \frac{\hbar}{2mi}(A\nabla + \nabla A)$. The measurement yields

$$\langle B\rangle = \frac{1}{v_n}\int_{V_n} \frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*)dv = \frac{1}{v_n}\int_{V_n} j(x)dv. \tag{6}$$

This is the average value of the flux density $j(x)$ in the region $V_n$. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions $V_n$ we can measure $\rho(x)$ and $j(x)$ everywhere in space. Since the wave function $\psi(x)$ can be uniquely expressed by $\rho(x)$ and $j(x)$ (except for an overall phase factor), the whole wave function of the measured system at a given instant can be measured by PMs.[2]

[2]There are two known schemes of PM. The first scheme is to introduce a protective potential such that the wave function of the measured system at a given instant, $|\psi\rangle$, is a nondegenerate energy eigenstate of the total Hamiltonian of the system with finite gap to neighboring energy eigenstates. By this scheme, the measurement of an observable is required to be weak and adiabatic. The second scheme is via the quantum Zeno effect. The Zeno effect is realized by making frequent projective measurements of an observable, of which the wave function of the measured system at a given instant, $|\psi\rangle$, is a nondegenerate eigenstate. By this scheme, the measurement of the measured observable is not necessarily weak but weaker than the Zeno projective measurements.

Now let's analyze possible implications of PMs for the meaning of the wave function and QBism.[3] First, since the result of a PM of an observable on a quantum system being in a superposition of different eigenstates of the observable is always definite, the superposed wave function does not represent probability assignments for PMs, no matter these probabilities are objective or subjective. Next, the wave function can be measured by a series of PMs on a single quantum system. This means that the wave function is a representation of the objective outcome of the interaction between the measured system and the measuring device during the PMs, such as the shift of the pointer of a measuring device which makes the PMs.[4] For example, the modulus squared of the wave function in position $x$, $|\psi(x)|^2$, is a representation of the result of the interaction between the measured system and the measuring device during a PM of the projection operator in $x$. Since the result of a PM being the modulus squared of the wave function is objective and definite, the wave function cannot be subjective degrees of belief of an agent, which may be different for different agents.

We can also reach the same conclusion by a somewhat different argument. According to QBism, the wave function represents subjective degrees of belief of an agent, and thus it is a property of an agent, not of the external world. Then, if the wave function can be measured, it can only be measured from the agent, not from the external world.[5] But PMs show that the wave function can be measured by a certain interaction between a quantum system and a measuring device, which are independent of any agent. Thus QBism is not consistent with PMs.

To sum up, for PMs, the wave function represents the objective results of the interactions between the measured system and the measuring device, and it does not represent probability assignments, either objective or subjective. This is against QBism, according to which the wave function represents

[3]Proponents of PMs argue that since PMs can measure the expectation values of observables and even the wave function on a single quantum system, they provide strong supports for the reality of the wave function, while some others disagree, and $\psi$-epistemic models have also been proposed to account for PMs (Combes et al, 2018). Recently I showed that although these $\psi$-epistemic models can explain the appearance of expectation values of observables in a single measurement, their predictions are different from those of quantum mechanics for some PMs. Moreover, I gave a proof of the reality of the wave function in terms of PMs under an auxiliary finiteness assumption about the dynamics of the ontic state (Gao, 2020). However, the new proof is still based on the ontological models framework that assumes the existence of the underlying ontic state of a quantum system. Thus it has no implications for QBism which denies this assumption.

[4]But this does not mean that the wave function must be a direct representation of the ontic state of the measured system (Combes et al, 2018). More work still needs to be done here (Gao, 2020).

[5]Here there is an interesting idea for QBism. If the wave function is a property of an agent, then it should be in principle measurable by measuring the brain state of the agent. This also means that the wave function in QBism is a property of a single system, not of an ensemble.

an agent's personal probability assignments, reflecting his subjective degrees of belief about the future content of his experience such as the result of a measurement. Since the above argument does not assume that there are agent-independent elements of physical reality that determine either measurement outcomes or probabilities of measurement outcomes, it is stronger than the $\psi$-ontology theorems in some sense, which can be proved only under this realist assumption.

As pointed out before (Gao, 2017), there are in fact two connections between the mathematical formalism of quantum mechanics and experience. The first connection is the well-known Born rule, and the connection is in general probabilistic. The second connection is PMs, and the connection is definite, determined only by the deterministic Schrödinger equation and independently of the Born rule. Then, the inconsistency between QBism and PM may be understandable. QBism refers only to the first connection, not to the second connection; it aims to interpret the wave function in the Born rule, but it ignores the wave function in PMs.

Finally, it is worth noting that the above no-go result is also valid for other pragmatist approaches to quantum theory which deny that the theory offers a description or representation of the physical world (Healey, 2017). One way to avoid the result is to deny the reality of measurement results. This is a radical way out, which will arguably lead to solipsism.

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